There is a useful empirical relationship between viscosity and flow rate (velocity):

$$Flow\ Rate\ (V_N) = \rho g d^2 \frac{\sin(\theta)}{3\eta} \tag{1}$$

ρ: flow density, g: gravitational acceleration, d: flow depth, η: viscosity, θ: slope

In simple fluids such as water, the rate of deformation, the strain rate, is directly proportional to the applied stress. Fluids with this property are called **Newtonian Fluids**. The ratio of the stress to the strain rate is called the Newtonian viscosity of the fluid.
Most lava flows are not simple liquids, but contain phenocrysts and gas bubbles. Consequently, they do not behave as Newtonian Fluid. They have a threshold value that must be overcome before the flow will move. This means that the stress applied to it must exceed a certain level called the yield strength before the lava will even begin to deform and flow. This type of fluid is known as **Bingham Fluid** [1] [2].

The shearing stress τ is given by:

$$\tau = d\rho g \sin(\theta) \tag{2}$$

And the yield strength of the flow is:

$$\tau_0 = h_0 \rho g \sin(\theta) \tag{3}$$

When h_0 is the critical flow thickness needed to achieve flow, and τ_0 is the yield strength. One estimate of the average flow velocity is:

$$V_B = \frac{\rho g d^2 \sin(\theta)}{3\eta}[1 - \frac{3\tau_0}{2\tau} + \frac{1}{2}(\frac{\tau_0}{\tau})^3] \tag{4}$$

By simplifying shearing stress and yield strength equation, we would have this relation:

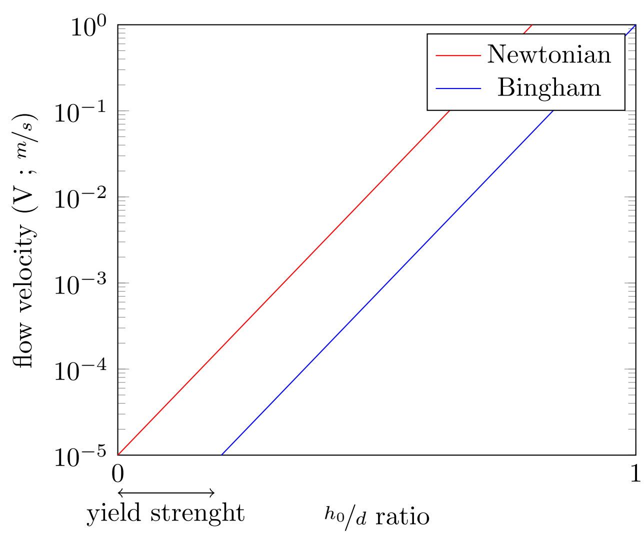
$$\frac{\tau_0}{\tau} = \frac{h_0}{d} \tag{5}$$

So we can rewrite Bingham fluid speed again but this time related to the ratio of h_0/d.

$$V_B = \frac{\rho g d^2 \sin(\theta)}{3\eta}[1 - \frac{3h_0}{2d} + \frac{1}{2}(\frac{h_0}{d})^3] \tag{6}$$

Now we can relate the speed of flow to its depth. We know that lava can only move if its depth (d) is greater than the critical thickness (h_0). Therefore, this ratio h_0/d is between 0 and 1. Flow velocity ranges from 10^{-5} (ms^{-1}) for silicic flows, 0.01-1 (ms^{-1}) for basaltic with tubes/channels (similar to the current case that we discuss here) and 0.1-5 (ms^{-1}) for basaltic sheet flows [3].

We can visualize the difference between Newtonian and Bingham flow speed over the h_0/d ratio like this:



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